CBSE SAMPLE PAPER - 05

Class 11 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The general solution of the equation tanθ = tanα is **[1]**

a) $\theta = 2n\pi \pm \alpha, n \in I$
b) $\theta = n\pi \pm \alpha, n \in I$
c) $\theta = n\pi + \alpha, n \in I$
d) $\theta = 2n\pi + \alpha, n \in I$

2. The algebraic sum of the deviations of 20 observations measured from 30 is 2. What would be the mean of the observations? **[1]**

a) 32
b) 30.1
c) 30
d) 30.2

3. For any two events A and B, which one of the following holds? **[1]**

a) P(A ∩ B) ≤ P(B) ≤ P(A) + P(B) ≤ P(A ∪ B)
b) P(A ∪ B) ≤ P(A) ≤ P(A ∩ B) ≤ P(A) + P(B)
c) P(A ∩ B) ≤ P(A) ≤ P(A ∪ B) ≤ P(A) + P(B)
d) P(A ∪ B) ≤ P(B) ≤ P(A ∩ B) ≤ P(A) + P(B)

4. $\lim\limits_{x \to 1} \frac{\sin \pi x}{x-1}$ is equal to **[1]**

a) $\frac{1}{\pi}$
b) π
c) $-\pi$
d) $-\frac{1}{\pi}$

5. A point equidistant from the lines 4x + 3y + 10 = 0, 5x – 12y + 26 = 0 and 7x + 24y – 50 = 0 is **[1]**

a) (0, 0)
b) (1, -1)
c) (1, 1)
d) (0, 1)

6. Two sets A and B are as under: **[1]**

$A = \{(a, b) \in R \times R: |a - 5| < 1 \text{ and } |b - 5| < 1\}$; $B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then

a) $B \subset A$

b) $A \cap B = \phi$ (an empty set)

c) $A \subset B$

d) neither $A \subset B$ nor $B \subset A$

7. The complex number z such that $\left|\frac{z-i}{z+i}\right| = 1$ lies on **[1]**

a) a circle

b) None of these

c) The x-axis

d) The line y = 1

8. If [x] denotes the greatest integer $\leq$ x, then $\left[\frac{2}{3}\right] + \left[\frac{2}{3} + \frac{1}{99}\right] + \left[\frac{2}{3} + \frac{2}{99}\right] + \ldots + \left[\frac{2}{3} + \frac{98}{99}\right] =$ **[1]**

a) 99

b) 66

c) 98

d) 65

9. Solve the system of inequalities $-2 < 1 - 3x < 7$ **[1]**

a) $-1 < x < 1$

b) none of these

c) $-2 < x < 2$

d) $-2 < x < 1$

10. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B)$ is equal to **[1]**

a) 2

b) 0

c) 1

d) 3

11. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to **[1]**

a) 720

b) 7200

c) 120

d) 60

12. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is **[1]**

a) 4

b) $\frac{-4}{5}$

c) $\frac{1}{5}$

d) None of these

13. The expression $\left(x + \sqrt{x^2 - 1}\right)^5 + \left(x - \sqrt{x^2 - 1}\right)^5$ is a polynomial of degree **[1]**

a) 6

b) 5

c) 20

d) 10

14. Solve: $3x + 5 < x - 7$, when x is a real number **[1]**

a) none of these

b) $x < -12$

c) $x < -6$

d) $x > -6$

15. In a town of 10,000 families it was found that 40% families buys newspaper A, 20% buy newspaper B, and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% buy all the three newspapers, then number of families which buy A only is **[1]**

a) 3300

b) 3400

c) 2900

d) 3100

16. $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = ?$ **[1]**

a) tan 52°

b) tan 37°

c) None of these

d) tan 8°

17. If $\frac{1-ix}{1+ix}$ = a + ib, then $a^2 + b^2$ = **[1]**

a) 0

b) 1

c) None of these

d) - 1

18. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is **[1]**

a) $^{20}C_9$

b) $^{16}C_{11}$

c) $^{16}C_5$

d) $^{16}C_9$

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**
Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Domain and Range of a relation R = {(x, y): x - 3y = 0} defined on the set A = {1, 2, 3} are respectively {1, 2, 3} and {2, 4, 6}. **[1]**
Reason (R): Domain and Range of a relation R are respectively the sets {a: a ∈ A and (a, b) ∈ R} and {b: b ∈ A and (a, b) ∈ R}

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the domain of the function, $f(x) = \sqrt{3-x} + \frac{1}{\sqrt{x^2-1}}$ **[2]**

22. If y= $\cos^2 x^2$, find $\frac{dy}{dx}$. **[2]**

23. Write the equation of the parabola with focus (0, 0) and directrix x + y - 4 = 0. **[2]**

OR

If the lines 2x - 3y = 5 and 3x - 4y = 7 are the diameters of a circle of area 154 sq units, then find the equation of the circle.

24. State whether A = {x : x is a letter in the word LOYAL} and B = {x: x is a letter of the word ALLOY} are equal? Justify your answer. **[2]**

25. Reduce the equation 3x - 2y + 4 = 0 to intercepts form and find the length of the segment intercepted between the axes. **[2]**

Section C

26. If $f(x) = \frac{1}{(1-x)}$ then show that f[f{f(x)}] = x. **[3]**

27. Verify that (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram. **[3]**

OR

Find the equation of the set of the point P, the sum of whose distance from A(4, 0, 0) and B(-4, 0, 0) is equal to 10.

28. Simplify $(x + 2y)^8 + (x - 2y)^8$ **[3]**

OR

Using binomial theorem, prove that (2^{3n} - 7n - 1) is divisible by 49, where $n \in N$

29. Find the real values of x and y, if (1 + i) (x + iy) = 2 - 5z. **[3]**

OR

Convert the complex number in the polar form: $\sqrt{3}+i$

30. Solve the inequality 3 (2 – x) $\geq$ 2 (1 – x) for real x. **[3]**

31. In how many ways can 9 examination papers be arranged so that the best and the worst papers are never together? **[3]**

Section D

32. Two dice are thrown. The events A, B and C are as follows: **[5]**

A: getting an even number on the first die

B: getting on odd number on the first die

C: getting the sum of the numbers on the dice $\leqslant 5$

Describe the events

(i) A' (ii) not B (iii) A or B (iv) A and B (v) A but not C (vi) B or C (vii) B and C (viii) $A \cap B' \cap C'$

33. Show that $\lim\limits_{x \to \infty}\left(\sqrt{x^2+x+1}-x\right) \neq \lim\limits_{x \to \infty}\left(\sqrt{x^2+1}-x\right)$. **[5]**

OR

Evaluate: $\lim_{x \to 0} \frac{1-\cos x \cos 2x \cos 3x}{\sin^2 2x}$

34. Verify whether tan x tan$\left(x+\frac{\pi}{3}\right)$ + tan x tan$\left(\frac{\pi}{3}-x\right)$ + tan$\left(x+\frac{\pi}{3}\right)$ tan$\left(x-\frac{\pi}{3}\right)$ = -3. **[5]**

OR

Prove that: $\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}=\frac{1}{64}$.

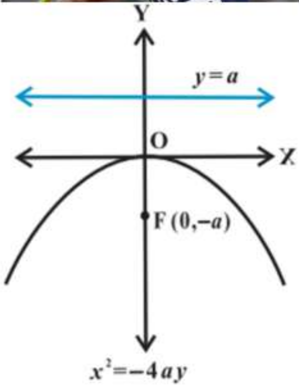
35. Find the variance and standard deviation for the following distribution. **[5]**

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) Name the shape of path followed by a javelin. If equation of such a curve is given by x^2 = -16y, then find the coordinates of foci.

(ii) Find the equation of directrix and length of latus rectum of parabola $x^2 = -16y$.

(iii) Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix.

OR

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

Father of Ashok is a builder, He planned a 12 story building in Gurgaon sector 5. For this, he bought a plot of 500 square yards at the rate of ₹ 1000 /yard². The builder planned ground floor of 5 m height, first floor of 4.75 m and so on each floor is 0.25 m less than its previous floor.

(i) What is the height of the last floor?

a) 2.5 m
b) 3 m
c) 2.75 m
d) 2.25 m

(ii) Which floor no is of 3 m height?

a) 10
b) 7
c) 5
d) 9

(iii) What is the total height of the building?

a) 40.5 m
b) 44 m
c) 40 m
d) 43.5

OR

Up to which floor the height is 33 m?

a) 8
b) 7
c) 9
d) 10

38. **Read the text carefully and answer the questions:** **[4]**

The given table shows the 10 most famous engineering private colleges of India with their respective fee structure of 4 years Bachelor of Technology course. The Budget of Ram is 6 lakhs and budget of Sri is 4 lakhs.

Private College	Fee structure (4 Years course)
Birla Institute of Technology & Science	11.57 Lakhs
SRM University	9 Lakhs

Manipal Institute of Technology	7 Lakhs
AMITY University	10 Lakhs
Jaypee Institute of Information Technology	6 Lakhs
LPU University	6.8 Lakhs
Thapar Institute of Technology	8 Lakhs
Kalinga Institute of Technology	4 Lakhs
Vellore Institute of Technology	9 Lakhs
Chandigarh University	3.3 Lakhs

(i) Sri has taken an education loan of 3 lakhs to increase his overall budget. Find the set of college in which Ram can take admission but Sri cannot.

(ii) Ram has taken an education loan of 2 lakhs to increase his overall budget. Find the number of colleges in which he can take admission after raising his budget.

Solution

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Class 11 - Mathematics

Section A

1. **(b)** $\theta = n\pi \pm \alpha, n \in I$

Explanation: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in I$

$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \sin\theta \cos\alpha = \sin\alpha \cos\theta$

$\Rightarrow \sin\theta \cos\alpha - \sin\alpha \cos\theta = 0$

$\Rightarrow \sin(\theta - \alpha) = 0$

and sin x = 0

x = nπ, n $\in$ I

$\Rightarrow \theta - \alpha = n\pi, n \in I$

$\Rightarrow \theta = n\pi + \alpha, n \in I$

2. **(b)** 30.1

Explanation: According to the question, $\sum_{i=1}^{20}(x_i - 30)$ = 2 [given]

$\Rightarrow \sum_{i=1}^{20} x_i - 600 = 2 \Rightarrow \sum_{i=1}^{20} x_i = 602 \Rightarrow \text{Mean} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{602}{20} = 30.1$

3. **(c)** P(A $\cap$ B) $\leq$ P(A) $\leq$ P(A $\cup$ B) $\leq$ P(A) + P(B)

Explanation: Clearly, A $\cap$ B $\subseteq$ A

$\Rightarrow$ P(A $\cap$ B) $\leq$ P(A) ...(i)

A $\subseteq$ A $\cup$ B

$\Rightarrow$ P(A) $\leq$ P(A) + P(B) ...(ii)

We know that,

P(A $\cup$ B) + P(A) + P(B) - P(A $\cap$ B)

$\Rightarrow$ P(A $\cup$ B) $\leq$ P(A) + P(B) ...(iii)

From Eqs. (i), (ii), and (iii),

P(A $\cap$ B) $\leq$ P(A) $\leq$ P(A $\cup$ B) $\leq$ P(A) + P(B)

4. **(c)** $-\pi$

Explanation: $\lim_{x \to 1} \frac{\sin\pi x}{x-1}$

$= \lim_{h \to 0} \frac{\sin\pi(1+h)}{(1+h)-1}$

$= \lim_{h \to 0} \frac{\sin(\pi+\pi h)}{h}$

$= \lim_{h \to 0} \frac{-\sin\pi h}{h}$

$= \lim_{h \to 0} -\left(\frac{\sin\pi h}{\pi h}\right)\pi$

$= -\pi$

5. **(a)** (0, 0)

Explanation: We note that distance of each of three lines from (0, 0) is 2 units

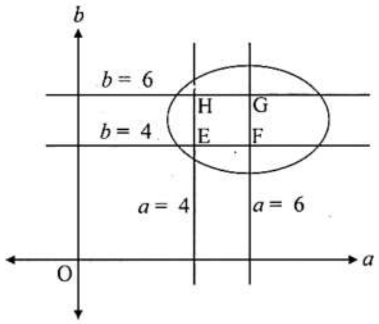
6. **(c)** A $\subset$ B

Explanation: |a - 5| < 1 and |b - 5| < 1

$\Rightarrow$ 4 < a < 6 and 4 < b < 6

$4(a - 6)^2 + 9(6 - 5)^2 \leq 36$

$\Rightarrow \frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$



Set A represents square EFGH and Set B represents an ellipse.

$\Rightarrow A \subset B$

7. **(c)** The x-axis

Explanation: $\left|\frac{z-i}{z+i}\right| = 1 \Rightarrow \left|\frac{z-i}{z+i}\right|^2 = 1$

$\Rightarrow \left|\frac{x+iy-i}{x+iy+i}\right|^2 = 1 \Rightarrow \left|\frac{x+i(y-1)}{x+i(y+1)}\right|^2 = 1 \Rightarrow \frac{|x+i(y-1)|^2}{|x+i(y+1)|^2}$

$\Rightarrow \frac{x^2+(y-1)^2}{x^2+(y+1)^2} \Rightarrow x^2 + (y-1)^2 = x^2 + (y+1)^2$

$\Rightarrow (y+1)^2 - (y-1)^2 = 0 \Rightarrow 4y = 0 \Rightarrow y = 0$

$\Rightarrow$ z lies on the x-axis

8. **(b)** 66

Explanation: Given expression $= \sum_{i=0}^{98}\left[\frac{2}{3} + \frac{i}{99}\right]$

$= \sum_{i=0}^{32}\left[\frac{2}{3} + \frac{i}{99}\right] + \sum_{i=33}^{98}\left[\frac{2}{3} + \frac{i}{99}\right]$

$= 0 + \sum_{i=33}^{98}\left[\frac{2}{3} + \frac{i}{99}\right]$ ... [$\because \frac{2}{3} \le \frac{2}{3} + \frac{i}{99} < 1$ for i = 0, 1, 2, ..., 32]

= 66 [$\because$ each term in the summation is one or more but less than 2 when i = 33, 34, 35, ... ,98]

9. **(d)** $-2 < x < 1$

Explanation: $-2 < 1 - 3x < 7$

$\Rightarrow -2 - 1 < 1 - 3x - 1 < 7 - 1$

$\Rightarrow -3 < -3x < 6$

$\Rightarrow \frac{-3}{-3} > \frac{-3x}{-3} > \frac{6}{-3}$

$\Rightarrow 1 > x > -2$

$\Rightarrow -2 < x < 1$

10. **(a)** 2

Explanation: $\tan(A - B) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\Rightarrow \tan A - \tan B = 1 + \tan A \tan B$ .... (i)

Now,

$(1 + \tan A)(1 - \tan B) = 1 + \tan A - \tan B - \tan A \tan B$

$= 1 + 1 + \tan A \tan B - \tan A \tan B$ (Using eq. (i))

= 2

11. **(b)** 7200

Explanation: Here, it is given that total numbers of vowels = 4

and total numbers of consonants = 5

Total number of words formed by 2 vowels and 3 consonants

$= {}^4C_2 \times {}^5C_3 = \frac{4!}{2!2!} \times \frac{5!}{3!2!} = \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} \times \frac{5 \times 4 \times 3!}{3! \times 2}$

$= 6 \times 10 = 60$

Now permutation of 2 vowels and 3 consonants = 5!

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Therefore,the total number of words = $60 \times 120 = 7200$

12. **(a)** 4

Explanation: We have,consider a G.P. a, ar, ar^2, ... with 2n terms. We have $\frac{a(r^{2n}-1)}{r-1} = \frac{5a((r^2)^n-1)}{r^2-1}$

(Since common ratio of odd terms will be r^2 and number of terms will be n)

$\Rightarrow \frac{a(r^{2n}-1)}{r-1} = 5\frac{a(r^{2n}-1)}{(r^2-1)}$

$\Rightarrow$ a (r + 1) = 5a, i.e., r = 4.

13. **(b)** 5

Explanation: $\left(x+\sqrt{x^2-1}\right)^5 + \left(x-\sqrt{x^2-1}\right)^5$

= $2[x^5 + {}^5C_2x^3\left(\sqrt{x^2-1}\right)^2 + {}^5C_4x^1\left(\sqrt{x^2-1}\right)^4]$

= $2x^5 + 2 \cdot {}^5C_2x^3(x^2 - 1) + 2 \cdot {}^5C_4\, x(x^2 - 1)^2$

which is a polynomial of degree 5.

14. **(c)** x < -6

Explanation: 3x + 5 < x - 7

$\Rightarrow$ 3x + 5 - x < x - 7 - x

$\Rightarrow$ 2x + 5 < -7

$\Rightarrow$ 2x + 5 - 5 < -7 - 5

$\Rightarrow$ 2x < -12

$\Rightarrow \frac{2x}{2} < -\frac{12}{2}$

$\Rightarrow$ x < -6

15. **(a)** 3300

Explanation: Let U denote the set of families who buy news paper and let X, Y and Z denote the sets of families who buy newspaper A, B and C respectively

Then we have n(U) = 10000, n(X) = 40% of 10000 = 4000, n(Y) = 20% of 10000 = 2000 and n(Z) = 10% of 10000 = 1000

Also n(X ∩ Y) = 5% of 10000 = 500, n(Y ∩ Z) = 3% of 10000 = 300

n(X ∩ Z) = 4% of 10000 = 400 and (X ∩ Y ∩ Z) = 2% of 10000 = 200

Hence the number of families who buy only newspaper A = n[X - (Y ∪ Z)]

= n(X) - n[X ∩(Y ∪ Z)]

= n(X) - n[X ∩(Y ∪ Z)]

= n(X) - [n(X ∩ Y) + n(X ∩ Z) - n((X ∩ Y) ∩(X ∩ Z))]

= n(X) - [n(X ∩ Y) + n(X ∩ Z) - n(X ∩ Y ∩ Z)]

= 4000 - [500 + 400 - 200]

= 4000 - 700 = 3300

16. **(b)** tan 37°

Explanation: $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \frac{1-\tan 8^\circ}{1+\tan 8^\circ} = \frac{\tan 45^\circ - \tan 8^\circ}{1+\tan 45^\circ \tan 8^\circ}\ [\because 1 = \tan 45^\circ]$

= tan (45° - 8°) = tan 37°

17. **(b)** 1

Explanation: 1

$\frac{1-ix}{1+ix}$ = a + ib

Taking modulus on both the sides, we get:

$\left|\frac{1-ix}{1+ix}\right| = |a+ib|$

$\Rightarrow \frac{\sqrt{1^2+x^2}}{\sqrt{1^2+x^2}} = \sqrt{a^2+b^2}$

$\Rightarrow \sqrt{a^2+b^2} = 1$

Squaring both the sides, we get:

$a^2 + b^2 = 1$

18. **(d)** ${}^{16}C_9$

Explanation: Total number of players = 22

2 players are always included and 4 are always excluding or never included = 22 - 2 - 4 = 16

∴ Required number of selection = ${}^{16}C_9$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + \ldots + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + \ldots (-1)^n n_{c_n}$

Each term will cancel each other

∴ $(1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20. **(d)** A is false but R is true.

Explanation: Assertion false because $x - 3y = 0 \Rightarrow y = \frac{x}{3} \Rightarrow y = \frac{1}{3}, \frac{2}{3}, 1$

so range = $\{\frac{1}{3}, \frac{2}{3}, 1\}$ and by definition reason is true.

Section B

21. Here we are given that, $f(x) = \sqrt{3-x} + \frac{1}{\sqrt{x^2-1}}$

Clearly, f(x) is defined for all real values of x for which

$3 - x \geq 0$ and $x^2 - 1 > 0$

$x - 3 \leq 0$ and $(x + 1)(x - 1) > 0$

$\Rightarrow x \leq 3$ and $(x < -1$ or $x > 1)$

$\Rightarrow (x \leq 3$ and $x < -1)$ or $(x \leq 3$ and $x > 1)$

$\Rightarrow (x < -1)$ or $(1 < x \leq 3)$

$\Rightarrow x \in (-\infty, -1) \cup (1, 3]$.

∴ dom (f) = $(-\infty, -1) \cup (1, 3]$

22. We have, $y = (\cos x^2)^2$

put $x^2 = t$ and $\cos x^2 = \cos t = u$, so that $y = u^2$

Therefore,we have,

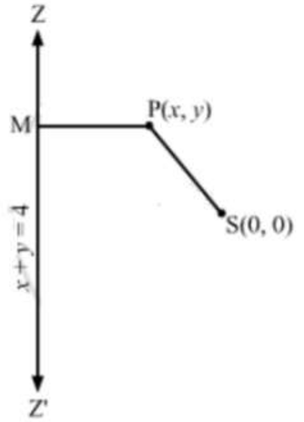
$\frac{dy}{du} = 2u, \frac{du}{dt} = -\sin t$ and $\frac{dt}{dx} = 2x$

So, $\frac{dy}{dx} = \left(\frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}\right)$

$= -4u x \sin t = -4x \sin t \cos t$

$= -4x \sin x^2 \cos x^2 = -2x \sin(2x^2)$

23. Let P (x, y) be any point on the parabola whose focus is S(0, 0) and the directrix is x + y = 4



Now draw PM perpendicular to x + y = 4

Therefore, we have: SP = PM

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = \left(\frac{x+y-4}{\sqrt{1+1}}\right)^2$

$\Rightarrow x^2 + y^2 = \left(\frac{x+y-4}{\sqrt{2}}\right)^2$

$\Rightarrow 2x^2 + 2y^2 = x^2 + y^2 + 16 + 2xy - 8y - 8x$

$\Rightarrow x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$

Which is the required equation of parabola.

OR

Given: $2x - 3y = 5$ and $3x - 4y = 7$

On solving these equations, we get x = 1, y = -1

So, centre of the circle is (1, -1)

Since, area of circle = πr^2

$\therefore 154 = \frac{22}{7}r^2$

$\Rightarrow r^2 = 7^2 \Rightarrow r = 7$

$\therefore$ Equation of circle is

$(x - 1)^2 + (y + 1)^2 = 7^2$

$\Rightarrow x^2 - 2x + 1 + y^2 + 2y + 1 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

24. We have, A = {L ,0 ,Y ,A ,L} = {L ,0 ,Y , A}

and, B = {A ,L ,L ,0 ,Y } = {L ,0 ,Y ,A }

Clearly, A = B.

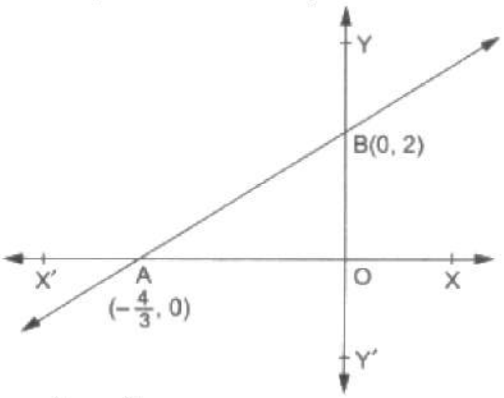
25. Here, it is given that

$3x - 2y + 4 = 0 \Rightarrow 3x - 2y = -4$

$\Rightarrow \frac{3x}{-4} + \frac{y}{2} = 1$ [on dividing both sides by - 4]

$\Rightarrow \frac{x}{\left(\frac{-4}{3}\right)} + \frac{y}{2} = 1$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{-4}{3}$ and b = 1



$\therefore \frac{x}{\left(\frac{-4}{3}\right)} + \frac{y}{2} = 1$ is the required equation in intercepts form.

Where ,x-intercept $= \frac{1}{2}$ and y-intercept = 2

If AB is part of the line intercepted between the axes then the end points of this line segment are $A\left(\frac{-4}{3}, 0\right)$ and B(0, 2).

Length AB $= \sqrt{\left(\frac{-4}{3} - 0\right)^2 + (0 - 2)^2} = \sqrt{\frac{16}{9} + 4} = \frac{2}{3}\sqrt{13}$ units.

Section C

26. We have,

$f(x) = \frac{1}{1-x}$

Now,

$f\{f(x)\} = f\left\{\frac{1}{1-x}\right\}$

$= \frac{1}{1 - \frac{1}{1-x}}$

$= \frac{1}{\frac{1-x-1}{1-x}}$

$= \frac{1-x}{-x}$

$= \frac{x-1}{x}$

$\therefore f[f\{x\}] = f\left\{\frac{x-1}{x}\right\}$

$= \frac{1}{1-\left(\frac{x-1}{x}\right)}$

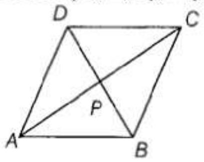
$= \frac{1}{\frac{x-x+1}{x}}$

$= \frac{x}{1}$

= x

$\therefore$ f[f(x)] = x Hence, proved.

27. Let A (-1, 2, 1), B(1, - 2, 5), C(4, - 7, 8) and D (2, - 3 , 4) are the vertices of a quadrilateral ABCD.



Then, mid-point of

$AC = \left(\frac{-1+4}{2}, \frac{2-7}{2}, \frac{1+8}{2}\right) = \left(\frac{3}{2}, \frac{-5}{2}, \frac{9}{2}\right)$ $\left[\because \text{coordinates of mid-point } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$

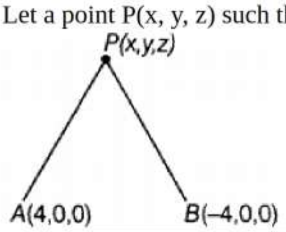
Similarly, mid-point of BD $= \left(\frac{3}{2}, -\frac{5}{2}, \frac{9}{2}\right)$

Mid-points of both the diagonals are the same (i.e., they bisect each other).

Hence, ABCD is a parallelogram.

OR

Let a point P(x, y, z) such that PA + PB = 10



$\Rightarrow \sqrt{(x-4)^2+(y-0)^2+(z-0)^2}+\sqrt{(x+4)^2+(y-0)^2+(z-0)^2}=10$

[$\because$ distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$]

$\Rightarrow \sqrt{x^2-8x+16+y^2+z^2}+\sqrt{x^2+8x+16+y^2+z^2}=10$

$\Rightarrow \sqrt{x^2+y^2+z^2-8x+16}= 10-\sqrt{x^2+y^2+z^2+8x+16}$

On squaring sides, we get

$x^2 + y^2 + z^2 - 8x + 16 = 100 + x^2 + y^2 + z^2 + 8x + 16$

$-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow -16x-100=-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow \quad 4x+25=5\sqrt{x^2+y^2+z^2+8x+16}$ [dividing both sides by -4]

Again squaring on both sides, we get

$16x^2 + 200x + 625 = 25(x^2 + y^2 + z^2 + 8x + 16)$

$\Rightarrow 16x^2 + 200x + 625 = 25x^2 + 25y^2 + 25z^2 + 200x + 400$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

28. $(x + 2y)^8 + (x - 2y)^8 = 2[{}^8C_0x^8+{}^8C_2x^5(2y)^2+{}^8C_4x^4(2y)^4+{}^8C_6x^2(2y)^6+{}^8C_8(2y)^8]$

$\because (x+a)^n+(x-a)^n=2[{}^nC_0x^n+{}^nC_2x^{n-2}a^2+{}^nC_4x^{n-4}a^4+\ldots\ldots\ldots]$

$\Rightarrow (x+2y)^8+(x-2y)^8=2[x^8+28x^6\times 4y^2+70x^4\times 16y^4+28x^2\times 64y^6+256y^8]$

$=2[x^8+112x^6y^2+1120x^4y^4+1792x^2y^6+256y^8]$

OR

To prove: $(2^{3n} - 7n - 1)$ is divisible by 49, where $n \in N$

$(2^{3n} - 7n - 1) = (2^3)n – 7n - 1$

$= 8^n - 7n - 1$

$= (1 + 7)^n - 7n - 1$

Now using binomial theorem..

$\Rightarrow {}^nC_0 1^n + {}^nC_1 1^{n-1}7 + {}^nC_2 1^{n-2}7^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= {}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= 1 + 7n + 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}] - 7n - 1$

$= 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}]$

$= 49[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2}]$

$= 49K$, where $K = ({}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1} 7^{n-3} + {}^nC_n 7^{n-2})$

Now, $(2^{3n} - 7n - 1) = 49K$

Therefore $(2^{3n} - 7n - 1)$ is divisible by 49.

29. $(1 + i)(x + iy) = 2 - 5i$

$\Rightarrow 1(x + iy) + i(x + iy) = 2 - 5i$

$\Rightarrow x + iy + ix + i^2y = 2 - 5i$

$\Rightarrow x + iy + ix - y = 2 - 5i$

$\Rightarrow x - y + i(x + y) = 2 - 5i$

Equating real and imaginary parts we get

$x - y = 2$ ... (i)

$x + y = -5$ ... (ii)

Adding (i) and (ii) we get

$2x = 2 - 5$

$\Rightarrow 2x = -3$

$\Rightarrow x = \frac{-3}{2}$

Substituting the value of x in (i), we get

$\frac{-3}{2} - y = 2$

$\Rightarrow \frac{-3}{2} - 2 = y$

$\Rightarrow y = \frac{-3-4}{2}$

$\Rightarrow y = \frac{-7}{2}$

Hence

$x = \frac{-3}{2}, y = \frac{-7}{2}$

OR

Here $z = \sqrt{3} + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = \sqrt{3}$ and $r\ \sin\theta = 1$

Squaring both sides of (i) and adding

$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1 \Rightarrow r^2 = 4 \Rightarrow r = 2$

$\therefore 2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

Since $\sin\theta$ and $\cos\theta$ are both positive

$\therefore \theta$ lies in first quadrant

$\therefore \theta = \frac{\pi}{6}$

Hence polar form of z is $2\left(\cos\frac{\pi}{6} + i\ \sin\frac{\pi}{6}\right)$

30. Here $3(2 - x) \geqslant 2(1 - x)$

$\Rightarrow 6 - 3x \geqslant 2 - 2x$

$\Rightarrow -3x + 2x \geqslant 2 - 6$

$\Rightarrow -x \leqslant -4$

Dividing both sides by -1, we have

$\frac{-x}{-1} < \frac{-4}{-1} \Rightarrow x \leqslant 4$

Thus the solution set is $(-\infty, 4]$.

31. The number of arrangements in which the best and the worst papers never come together can be obtained by subtracting from the total number of arrangements, the number of arrangements in which the best and worst come together.

The total number of arrangements of 9 papers = ${}^9P_9 = 9!$

Considering the best and the worst papers as one paper, we have 8 papers which can be arranged in ${}^8P_8$ = 8! ways. But, the best and worst papers can be put together in 2! ways. So, the number of permutations in which the best and the worst papers can be put together = (2! × 8!).

Hence, the number of ways in which the best and the worst papers never come together = 9! - 2! × 8! = 9 × 8! - 2 × 8! = 7 × 8 ! = 282240

Section D

32. When two dice are thrown then

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

A: getting an even number on the first die.
= {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2,6),
(4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

B: getting an odd number on the first die
{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

C: getting the sum of the number on the dice $\leqslant 5$
= {(1, 1),(1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

(i) A' = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)} = B

(ii) not B = {(2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = A

(iii) A or B =$A \cup B$ = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, l), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = S

(iv) A and B = $A \cap B = \phi$

(v) A but not C = A - C = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(vi) B or C = $B \cup C$ = {(1,1), (1,2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (5,1), (5, 2),(5, 3), (5, 4), (5, 5), (5, 6)}

(vii) B and C = $B \cap C$ = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)}

(viii) $A \cap B' \cap C'$ = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

33. We have to show that

$$\lim_{x \to \infty}(\sqrt{x^2+x+1}-x) \neq \lim_{x \to \infty}(\sqrt{x^2+1}-x)$$

LHS:

$$\lim_{x \to \infty}((\sqrt{x^2+x+1}-x))$$

Rationalising the numerator:

$$\lim_{x \to \infty}\left[\frac{(\sqrt{x^2+x+1}-x)(\sqrt{x^2+x+1}+x)}{(\sqrt{x^2+x+1}+x)}\right]$$

$$= \lim_{x \to \infty}\left[\frac{(x^2+x+1)-x^2}{(\sqrt{x^2+x+1}+x)}\right]$$

$$= \lim_{x \to \infty}\left[\frac{x+1}{(\sqrt{x^2+x+1}+x)}\right]$$

Dividing the numerator and the denominator by x:

$$\lim_{x\to\infty}\left[\frac{1+\frac{1}{x}}{\frac{\sqrt{x^2+x+1}}{x}+1}\right]$$

$$= \lim_{x\to\infty}\left[\frac{1+\frac{1}{x}}{\sqrt{\frac{x^2+x+1}{x^2}}+1}\right]$$

$$= \lim_{x\to\infty}\left[\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}\right]$$

When $x \to \infty$, then $\frac{1}{x} \to 0$

$\frac{1}{\sqrt{1}+1}$

$= \frac{1}{2}$

RHS:

$\lim_{x\to\infty}(\sqrt{x^2+1}$ - x) [From $\infty - \infty$]

Rationalising the numerator:

$$\lim_{x\to\infty}\left[\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{(\sqrt{x^2+1}+x)}\right]$$

$$= \lim_{x\to\infty}\left[\frac{x^2+1-x^2}{(\sqrt{x^2+1}+x)}\right]$$

$= \frac{1}{\infty}$

= 0

Hence, $\lim_{x\to\infty}[\sqrt{x^2+x+1}-x] \neq \lim_{x\to\infty}(\sqrt{x^2+1}$ - x)

OR

Clearly,

cos x cos 2x cos 3x = $\frac{1}{2}$ {2 cos x cos 2x cos 3x}

= $\frac{1}{2}$ {(2 cos x cos 2x) cos 3x}

= $\frac{1}{2}$ {(cos 3x + cosx) cos 3x}

= $\frac{1}{2}$ {$\cos^2$ 3x + cos 3x cos x}

= $\frac{1}{4}$ {2 $\cos^2$ 3x + 2 cos 3x cos x}

= $\frac{1}{4}$ {1 + cos 6x + cos 4x + cos 2x}

$$\therefore \lim_{x\to 0}\frac{1-\cos x\ \cos 2x \cos 3x}{\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{1-\frac{1}{4}(1+\cos 6x+\cos 4x+\cos 2x)}{\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{4-1-\cos 6x-\cos 4x-\cos 2x}{4\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{(1-\cos 6x)+(1-\cos 4x)+(1-\cos 2x)}{4\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{2\sin^2 3x+2\sin^2 2x+2\sin^2 x}{4\sin^2 2x}$$

$$= \lim_{x\to 0}\frac{\frac{\sin^2 3x}{x^2}+\frac{\sin^2 2x}{x^2}+\frac{\sin^2 x}{x^2}}{2\left(\frac{\sin^2 2x}{x^2}\right)}$$

$$= \lim_{x\to 0}\frac{\left(\frac{\sin 3x}{x}\right)^2+\left(\frac{\sin 2x}{x}\right)^2+\left(\frac{\sin x}{x}\right)^2}{2\left(\frac{\sin 2x}{x}\right)^2}$$

$$= \lim_{x\to 0}\frac{9\times\left(\frac{\sin 3x}{3x}\right)^2+4\times\left(\frac{\sin 2x}{2x}\right)^2+\left(\frac{\sin x}{x}\right)^2}{2\times 4\left(\frac{\sin 2x}{2x}\right)^2}$$

$$= \frac{9\times 1+4\times 1+1}{8} = \frac{14}{8} = \frac{7}{4}$$

34. We have to show that whether tan x tan$\left(x+\frac{\pi}{3}\right)$ + tan x tan$\left(\frac{\pi}{3}-x\right)$ + tan$\left(x+\frac{\pi}{3}\right)$ tan$\left(x-\frac{\pi}{3}\right)$ = -3.

LHS = tan x tan$\left(x+\frac{\pi}{3}\right)$ + tan x tan$\left(\frac{\pi}{3}-x\right)$ + tan$\left(x+\frac{\pi}{3}\right)$ tan$\left(x-\frac{\pi}{3}\right)$

= tan$\left(\frac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}\right)$ + tan x$\left(\frac{\tan\frac{\pi}{3}-\tan x}{1+\tan x\tan\frac{\pi}{3}}\right)$ ... [$\because$ tan(A + B) = $\left(\frac{\tan A+\tan B}{1-\tan A\tan B}\right)$ and tan(A - B) =]$\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$

$= \tan\left(\frac{\tan x+\sqrt{3}}{1-\tan x(\sqrt{3})}\right) + \tan x\left(\frac{\sqrt{3}-\tan x}{1+\tan x(\sqrt{3})}\right) + \left(\frac{\tan x+\sqrt{3}}{1-\tan x(\sqrt{3})}\right)\left(\frac{\sqrt{3}-\tan x}{1+\tan x(\sqrt{3})}\right)$ (as $\tan\frac{\pi}{3} = \sqrt{3}$)

$= \left(\frac{\left(1+\tan x(\sqrt{3})\right)\tan x\left(\tan x+\sqrt{3}\right)+\left(1-\tan x(\sqrt{3})\right)\tan x\left(\sqrt{3}-\tan x\right)+\left(\tan x+\sqrt{3}\right)\left(\sqrt{3}-\tan x\right)}{\left(1-\tan x(\sqrt{3})\right)\left(1+\tan x(\sqrt{3})\right)}\right)$

$= \left(\frac{(1+\tan x(\sqrt{3}))\tan x(\tan x+\sqrt{3})+(1-\tan x(\sqrt{3}))\tan x(\sqrt{3}-\tan x)+(\tan x+\sqrt{3})(\sqrt{3}-\tan x)}{(1-\tan x(\sqrt{3}))(1+\tan x(\sqrt{3}))}\right)$

$= \left(\frac{(\tan x+\sqrt{3}\tan^2 x)(\tan x+\sqrt{3})+(\tan x-\sqrt{3}\tan^2 x)(\sqrt{3}-\tan x)+(\tan^2 x-3)}{\left(1-(\sqrt{3}\tan x)^2\right)}\right)$

$= \left(\frac{(\tan^2 x+\sqrt{3}\tan x+\sqrt{3}\tan^3 x+3\tan^2 x)+(\sqrt{3}\tan x-3\tan^2 x-\tan^2 x+\sqrt{3}\tan^3 x)+(\tan^2 x-3)}{\left(1-(\sqrt{3}\tan x)^2\right)}\right)$

$= \left(\frac{(0+2\sqrt{3}\tan x+2\sqrt{3}\tan^3 x+3\tan^2 x)+(\tan^2 x-3)}{\left(1-(\sqrt{3}\tan x)^2\right)}\right)$

$= \left(\frac{2\sqrt{3}\tan x+2\sqrt{3}\tan^3 x+4\tan^2 x-3}{(1-3\tan^2 x)}\right)$

$\neq -3$

Hence LHS $\neq$ RHS

OR

LHS $= \cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

Multiplying and Dividing by $2^6\sin\frac{\pi}{65}$

$= \frac{2^6\sin\frac{\pi}{65}\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= = \frac{2^5\left(2\sin\frac{\pi}{65}\cos\frac{\pi}{65}\right)\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$ ... $\{\because \sin 2x = 2\sin x\cos x\}$

$= \frac{2^5\sin\frac{2\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^4\left(2\sin\frac{2\pi}{65}\cos\frac{2\pi}{65}\right)\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^4\sin\frac{4\pi}{65}\cos\frac{4\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^3\left(2\sin\frac{4\pi}{65}\cos\frac{4\pi}{65}\right)\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^3\sin\frac{8\pi}{65}\cos\frac{8\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^2\left(2\sin\frac{8\pi}{65}\cos\frac{8\pi}{65}\right)\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2^2\sin\frac{16\pi}{65}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2\left(2\sin\frac{16\pi}{65}\cos\frac{16\pi}{65}\right)\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{2\sin\frac{32\pi}{65}\cos\frac{32\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{\sin\frac{64\pi}{65}}{2^6\sin\frac{\pi}{65}}$

$= \frac{\sin\left(\pi-\frac{\pi}{65}\right)}{2^6\sin\frac{\pi}{65}}$ ... $[\because \pi - \frac{\pi}{65} = \frac{65\pi-\pi}{65} = \frac{64\pi}{65}]$

$= \frac{\sin\frac{\pi}{65}}{2^5\sin\frac{\pi}{65}}$ ... $\{\because \sin(\pi-\theta) = \sin\theta\}$

$= \frac{1}{2^5} = \frac{1}{64}$

LHS = RHS

Hence Proved

35. We need to make the following table from the given data:

x_i	f_i	$d_i = x_i - 34.5$	$u_i = \frac{x_i-34.5}{10}$	u_i^2	f_iu_i	$f_iu_i^2$

4.5	1	-30	-3	9	-3	9
14.5	5	-20	-2	4	-10	20
24.5	12	-10	-1	1	-12	12
34.5	22	0	0	0	0	0
44.5	17	10	1	1	17	17
54.5	9	20	2	4	18	36
64.5	4	30	3	9	12	36
Total	N = 70				22	130

The formula to calculate the Variance is given as, $\sigma^2 = \left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right] \times h^2$

h = difference between $x_i - x_{i-1} = 10$

Substituting values from the table, variance is,

$= \left[\frac{130}{70} - \left(\frac{22}{70}\right)^2\right] \times 100 = \left[\frac{13}{7} - \left(\frac{11}{35}\right)^2\right] \times 100$

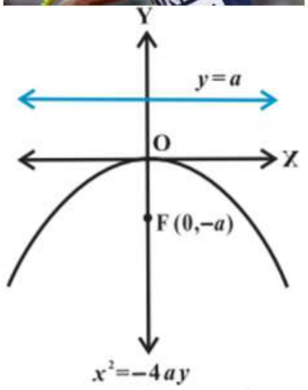
$= [1.857 - 0.099] \times 100 = 175.8$

and standard deviation = $\sqrt{Variance} = \sqrt{175.8} = 13.259$

Section E

36. **Read the text carefully and answer the questions:**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2 = -16y$ with $x^2 = -4ay$

$\Rightarrow -4a = -16$

$\Rightarrow a = 4$

coordinates of focus for parabola $x^2 = -4ay$ is (0, -a)

$\Rightarrow$ coordinates of focus for given parabola is (0, -4)

(ii) compare $x^2 = -16y$ with $x^2 = -4ay$

$\Rightarrow -4a = -16$

$\Rightarrow a = 4$

Equation of directrix for parabola $x^2 = -4ay$ is y = a

$\Rightarrow$ Equation of directrix for parabola $x^2 = -16y$ is y = 4

Length of latus rectum is 4a = 4 × 4 = 16

(iii) Equation of parabola with axis along y - axis

$x^2 = 4ay$

which passes through (5, 2)

$\Rightarrow 25 = 4a \times 2$

$\Rightarrow 4a = \frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

$\Rightarrow 2x^2 = 25y$

Equation of directrix is y= -a

Hence required equation of directrix is $8y + 25 = 0$.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is x = -2 and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with a = 2.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum = 4a = 8

37. **Read the text carefully and answer the questions:**

Father of Ashok is a builder, He planned a 12 story building in Gurgaon sector 5. For this, he bought a plot of 500 square yards at the rate of ₹ 1000 /yard². The builder planned ground floor of 5 m height, first floor of 4.75 m and so on each floor is 0.25 m less than its previous floor.

(i) **(d)** 2.25 m

Explanation: 2.25 m

(ii) **(d)** 9

Explanation: 9

(iii) **(d)** 43.5

Explanation: 43.5

OR

(a) 8

Explanation: 8

38. **Read the text carefully and answer the questions:**

The given table shows the 10 most famous engineering private colleges of India with their respective fee structure of 4 years Bachelor of Technology course. The Budget of Ram is 6 lakhs and budget of Sri is 4 lakhs.

Private College	Fee structure (4 Years course)
Birla Institute of Technology & Science	11.57 Lakhs
SRM University	9 Lakhs
Manipal Institute of Technology	7 Lakhs
AMITY University	10 Lakhs
Jaypee Institute of Information Technology	6 Lakhs
LPU University	6.8 Lakhs

Thapar Institute of Technology	8 Lakhs
Kalinga Institute of Technology	4 Lakhs
Vellore Institute of Technology	9 Lakhs
Chandigarh University	3.3 Lakhs

(i) On increasing the budget of Sri by 3 lakh, the total budget will be 7 lakhs. So, the set of college in which she can take admission are {Manipal, LPU, Jaypee, Chandigarh, Kalinga}. But the set of colleges in which Ram can take admission are {Manipal, LPU, Thapar, Jaypee, Chandigarh, Kalinga}. Since there is one element that is not common in both the sets, the required set is {Thapar}.

(ii) Earlier when the budget was 6 lakh, the set of colleges are {Jaypee, Chandigarh, Kalinga}. On increasing the budget by 2 lakhs, the total budget of Ram becomes 8 lakhs. So, the set of college in which he can take admission are {Manipal, LPU, Thapar, Jaypee, Chandigarh, Kalinga}.